USN | | | | | | | | | | **06CS/IS34**

Third Semester B.E. Degree Examination, June/July 2014

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO question from each part.***

PART – A

1 a. For any three sets A, B, C prove that $(A-B)-C = A-(B\cup C) = (A-C)-(B-C)$. **(06 Marks)**

b. A student visits an arcade each day after school and plays one game of either laser man, millipede or space conquerors. In how many ways can he play one game each day so that he plays each of the three types atleast once during a given school week (from Monday to Friday)? **(07 Marks)**

c. A girl rolls a fair die three times. What is the probability that :
i) Her second and third rolls are both larger than her first roll?
ii) The result of her second roll is greater than that of her first roll and the result of her third roll is greater than the second. **(07 Marks)**

2 a. Find the possible truth values of p, q and r in the following cases :
i) $p \to (q \vee r)$ is false ii) $p \wedge (q \to r)$ is true. **(05 Marks)**

b. Define tautology. Prove that, for any propositions p, q, r the compound proposition : $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(05 Marks)**

c. Simplify the following switching network using the laws of logic. **(05 Marks)**

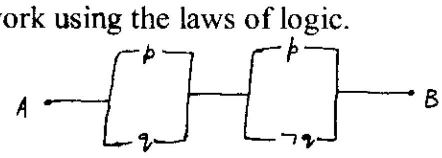

Fig. Q2(c)

d. Test the validity of the following argument :
I will become famous or I will not become a musician
I will become a musician
∴ I will become famous. **(05 Marks)**

3 a. Consider the following open statements with the set of all real numbers as the universe.
$p(x) : x \geq 0$, $q(x) : x^2 \geq 0$
$r(x) : x^2 - 3x - 4 = 0$
determine the truth values of the following statements :
i) $\forall x, p(x) \to q(x)$
ii) $\exists x, p(x) \wedge r(x)$
iii) $\forall x, r(x) \to p(x)$. **(06 Marks)**

b. Write down the negation of the following statements :
i) For all integers n, if n is not divisible by 2, then n is odd
ii) If k, m, n are any integers where $(k-m)$ and $(m-n)$ are odd, then $(k-n)$ is even. **(07 Marks)**

c. Give : i) a direct proof ii) an indirect proof, iii) proof by contradiction, for the following statement : "If n is an odd integer, then n + 11 is an even integer". **(07 Marks)**

4 a. Prove by mathematical induction that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(05 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_n = 2a_{n-1} + 1$ with $a_1 = 7$ for $n \geq 2$. **(05 Marks)**

c. Show that if any 5 numbers from 1 to 8 are chosen then two of them will have their sum equal to 9. **(05 Marks)**

d. Let S = {−1, 0, 2, 4, 7}. Find f(S) if
i) $f(x) = 1$ ii) $f(x) = 2x + 1$ iii) $f(x) = \left\lceil \frac{x}{5} \right\rceil$. **(05 Marks)**

PART – B

5 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by a R b if and only if 'a is a multiple of b'. Represent the relation R as a matrix and draw its digraph. **(06 Marks)**

b. On the set Z^+ of positive integers, a relation R is defined by a R b if and only if 'a divides b'. Prove that R is reflexive, transitive and antisymmetric, but not symmetric. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 5}. Define a relation R and A × A by (x_1, y_1) R (x_2, y_2) if and only if $x_1 + y_1 = x_2 + y_2$.

i) Verity that R is an equivalence relation on A × A

ii) Determine the equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)]. **(07 Marks)**

6 a. If R is an relation on the set A = {1,2 , 3, 4} defined by xRy if x/y, prove that (A, R) is a poset. Draw its Hasse diagram. **(05 Marks)**

b. Consider the Hasse diagram of a poset (A,R) as in Fig. Q6(b)

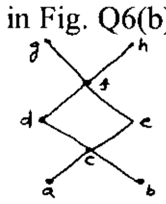

Fig. Q6(b)

If B = {c, d, e}, find :

i) All upper bounds of B

ii) All lower bounds of B

iii) The least upper bound of B

iv) The greatest lower, bound of B. **(05 Marks)**

c. Let A = {x/x is real and x ≥ - 1}, and B = {x/x is real and x ≥ 0}. Consider the function : $f: A \rightarrow B$ defined by $f(a) = a + 1$ for all $a \in A$ show that f is invertible and determine f^{1}. **(05 Marks)**

d. Let f, g, h be functions from z to z defined by :

$f(x) = x - 1$, $g(x) = 3x$

$$h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$$

Verify that $f_0(goh) = (fog)$ oh. **(05 Marks)**

7 a. Define ableian group. Prove that a group G is abelian if and only if $(a\ b)^2 = a^2b^2$ for all a, b ∈G. **(06 Marks)**

b. Define left and right costs. State and prove Lagrange's theorem. **(07Marks)**

c. Define homomorphism. Let f be a homomorphism from a group G_1 to a group G_2. Prove that : i) If e_1 is the identify in G_1 and e_2 is the identify in G_{2_1} then $f(e_1) = e_2$

ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$. **(07 Marks)**

8 a. A binary symmetric channel has probability ρ = 0.05 of incorrect transmission. if the word c = 011011101 is transmitted, what is the probability that i) single error occurs ii) a double error occurs iii) a triple error occurs. **(06 Marks)**

b. The generator matrix for an encoding function : $E: Z_2^3 \rightarrow Z_2^6$ is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

i) Find the code words assigned to 110 and 010

ii) Obtain the associated parity –check matrix. **(07 Marks)**

c. Prove that the set Z with binary operations ⊕ and ⊙ defined by

$x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a ring. **(07 Marks)**

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